# Modelos Lineares

## múltiplas preditoras

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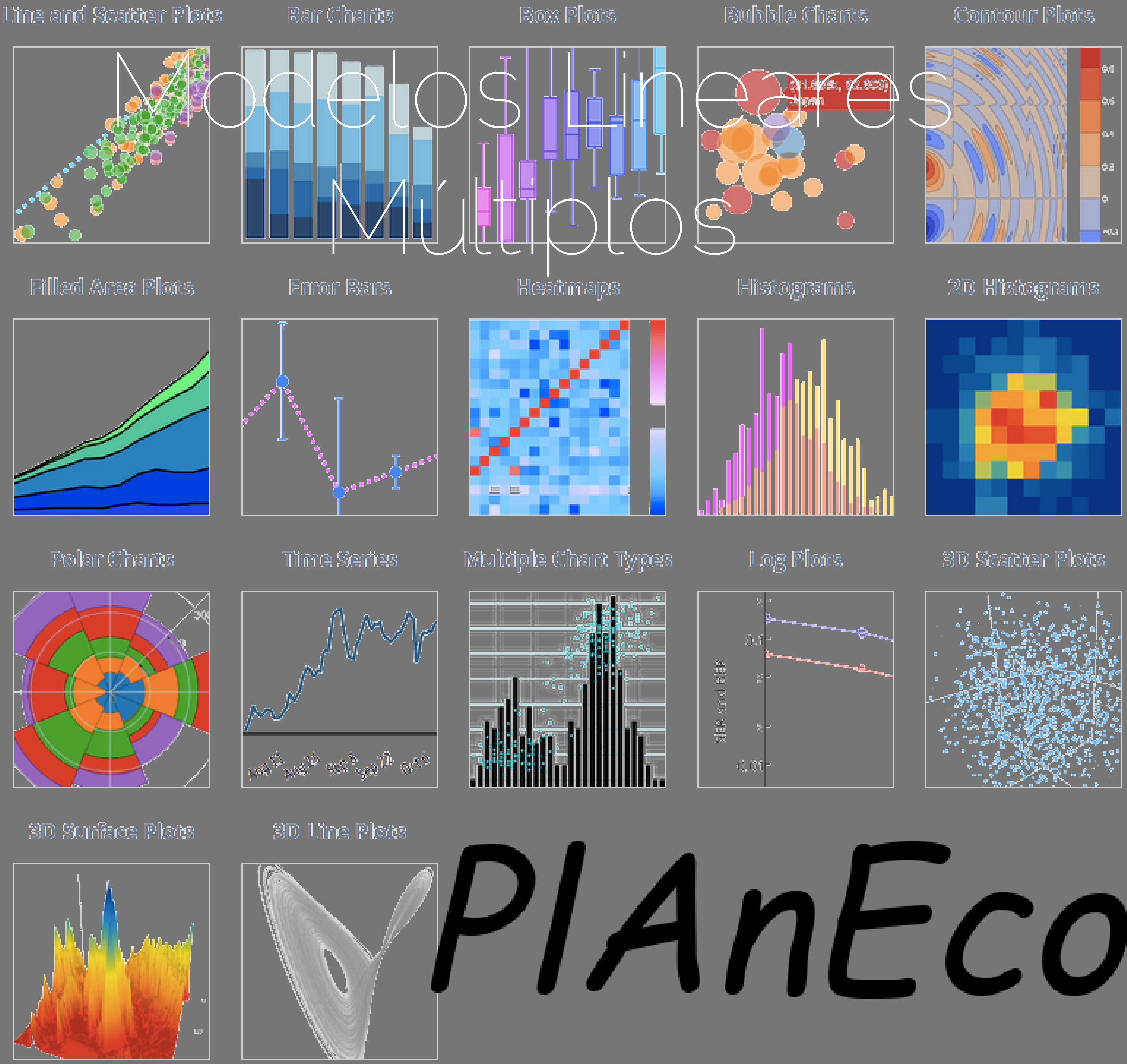
PlanECO 2018

# Modelos Lineares Múltiplos

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# Conceitos

## Conceitos

- preditoras contínuas e categóricas
- interação entre preditoras
- matriz do modelo (algebra linear)
- simplificação do modelo
- colinearidade

# Modelo Linear Simples

$$y = \alpha + \beta x + \epsilon$$
$$\epsilon = N(0, \sigma)$$

## Modelo Linear Múltiplo

$$y = \alpha + \sum \beta_i x_i + \epsilon$$
$$\epsilon = N(0, \sigma)$$

# Retomando o Modelo Linear

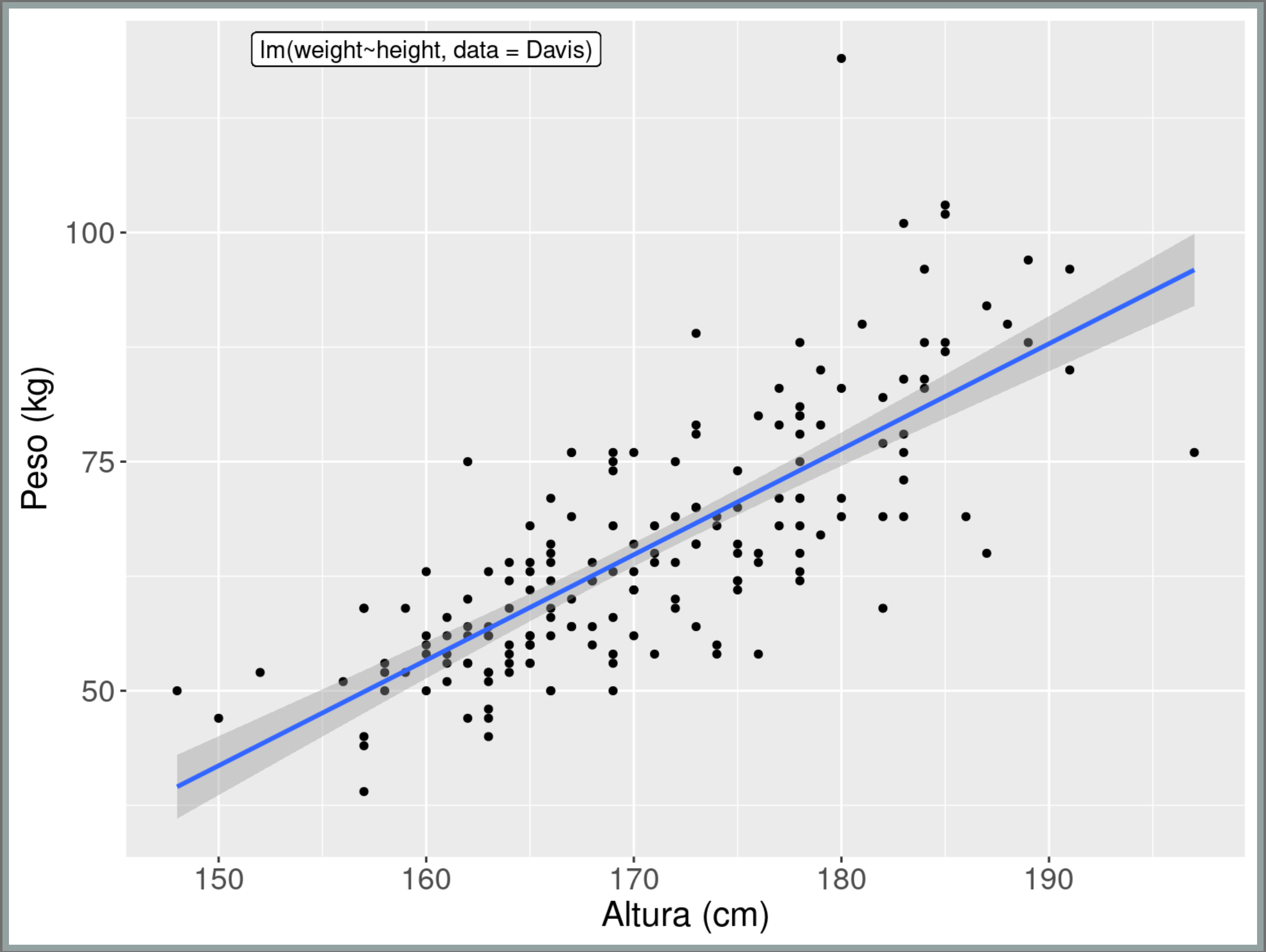
## Davis 1990

| | sex | weight | height | repwt | repht |
|---|---|---|---|---|---|
| 1 | M | 77 | 182 | 77 | 180 |
| 2 | F | 58 | 161 | 51 | 159 |
| 3 | F | 53 | 161 | 54 | 158 |
| 4 | M | 68 | 177 | 70 | 175 |
| 5 | F | 59 | 157 | 59 | 155 |
| 194 | F | 51 | 156 | 51 | 158 |
| 195 | F | 62 | 164 | 61 | 161 |
| 196 | M | 74 | 175 | 71 | 175 |
| 197 | M | 83 | 180 | 80 | 180 |
| 199 | M | 90 | 181 | 91 | 178 |
| 200 | M | 79 | 177 | 81 | 178 |

# Davis (1990)

| Variável | Descrição | Tipo |
| --- | --- | --- |
| sex | sexo | categórica dois nívei (M, F) |
| weight | peso | contínua (kg) |
| height | altura | contínua (cm) |
| repwt | peso reportado | contínua (kg) |
| repht | altura reportada | contínua (cm) |

# peso ~ weight



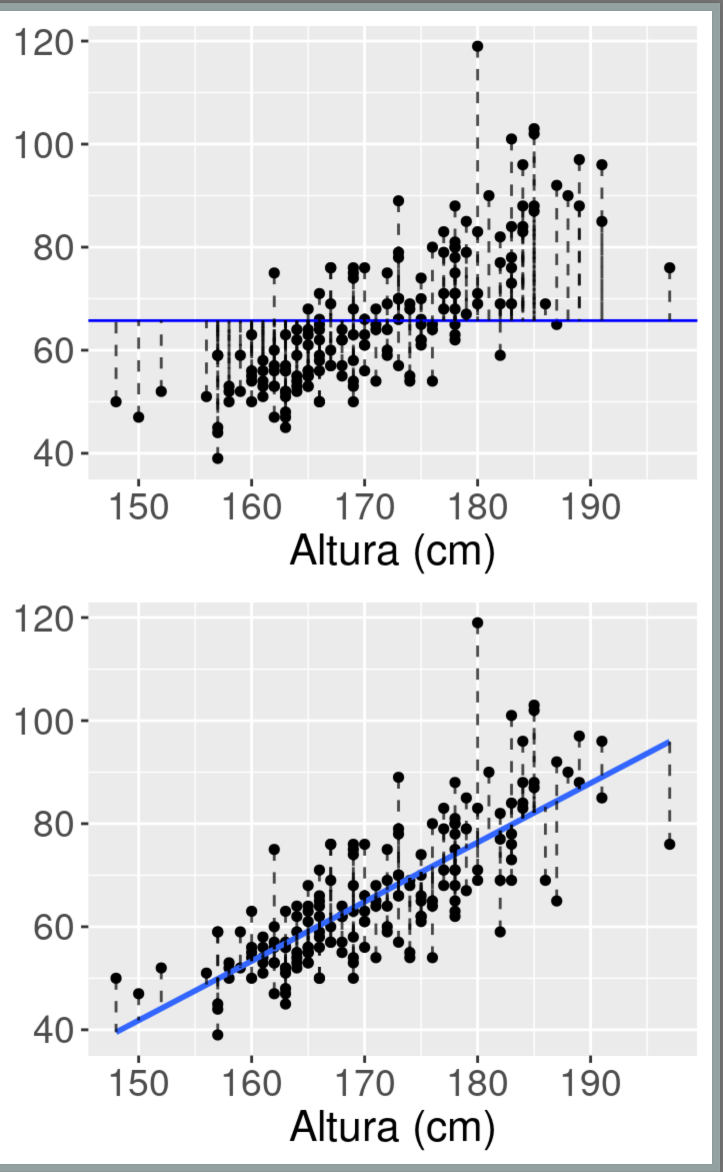
lm(weight~height, data = Davis)
100
75
50
150
160
170
180
190
Peso (kg)
Altura (cm)

# Modelo Linear

```
##
## Call:
## lm(formula = weight ~ height, data = Davis)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -19.928  -5.406  -0.651   4.891  42.641
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -130.84185   12.30184  -10.64   <2e-16 ***
## height         1.15112    0.07193   16.00   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.635 on 178 degrees of freedom
## Multiple R-squared:  0.5899, Adjusted R-squared:  0.5876
## F-statistic: 256.1 on 1 and 178 DF,  p-value: < 2.2e-16
```

```
## Analysis of Variance Table
##
## Response: weight
##            Df Sum Sq Mean Sq F value    Pr(>F)
## height      1  19095 19095.0  256.08 < 2.2e-16 ***
## Residuals 178  13273    74.6
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

# Modelo Linear: peso ~ altura




anova(

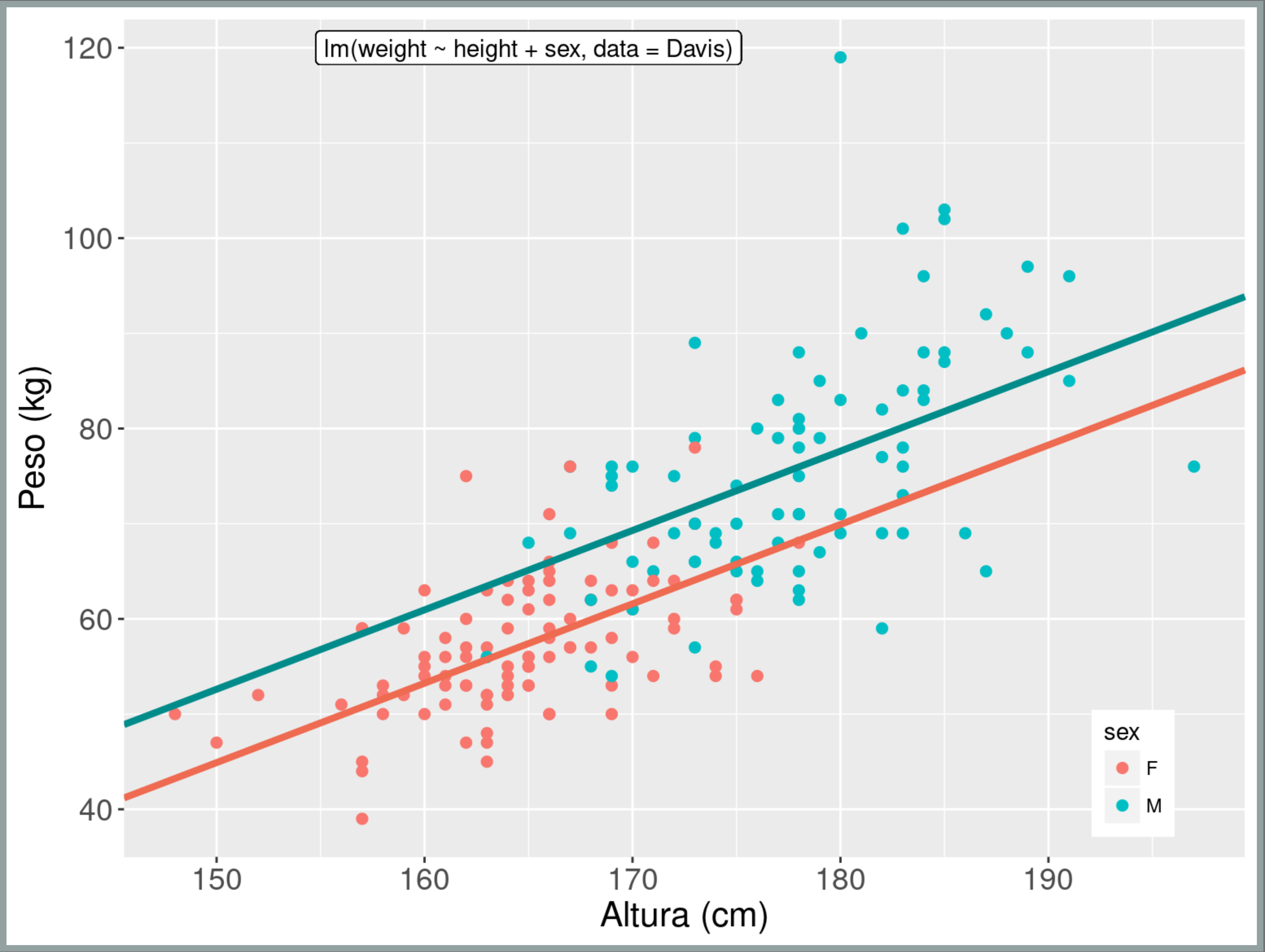
| Res.Df | RSS | Df |
|---|---|---|
| 179 | 32367.75 | |
| 178 | 13272.71 | 1 |

$p$

$p_{va}$

# Modelo Linear:

```
lm(weight ~ height + sex, data = Davis)
```


lm(weight ~ height + sex, data = Davis)
120
100
80
60
40
Peso (kg)
150
160
170
180
190
Altura (cm)
sex
F
M

# Resumo do Modelo

sexo: variável dummy com dois níveis (mulher = 0, homem = 1)

```
##
## Call:
## lm(formula = weight ~ height + sex, data = Davi
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -20.302  -4.808  -0.335   5.239  41.366
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|
## (Intercept) -80.2107    16.8415  -4.763 3.96e-0
## height        0.8341     0.1021   8.169 5.71e-1
## sexM          7.7070     1.8345   4.201 4.20e-0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
##
## Residual standard error: 8.258 on 177 degrees o
## Multiple R-squared:  0.6271, Adjusted R-squared
## F-statistic: 148.8 on 2 and 177 DF,  p-value: <
```

# Interpretando o modelo

lm(weight ~ height + sex, data = Davis)

```
## (Intercept)      height        sexM
## -80.2107328   0.8340964   7.7070166
```

Mulher (sex = 0)

$$w_f = \hat{\alpha} + \hat{\beta}_s sex + \hat{\beta}_h * height$$

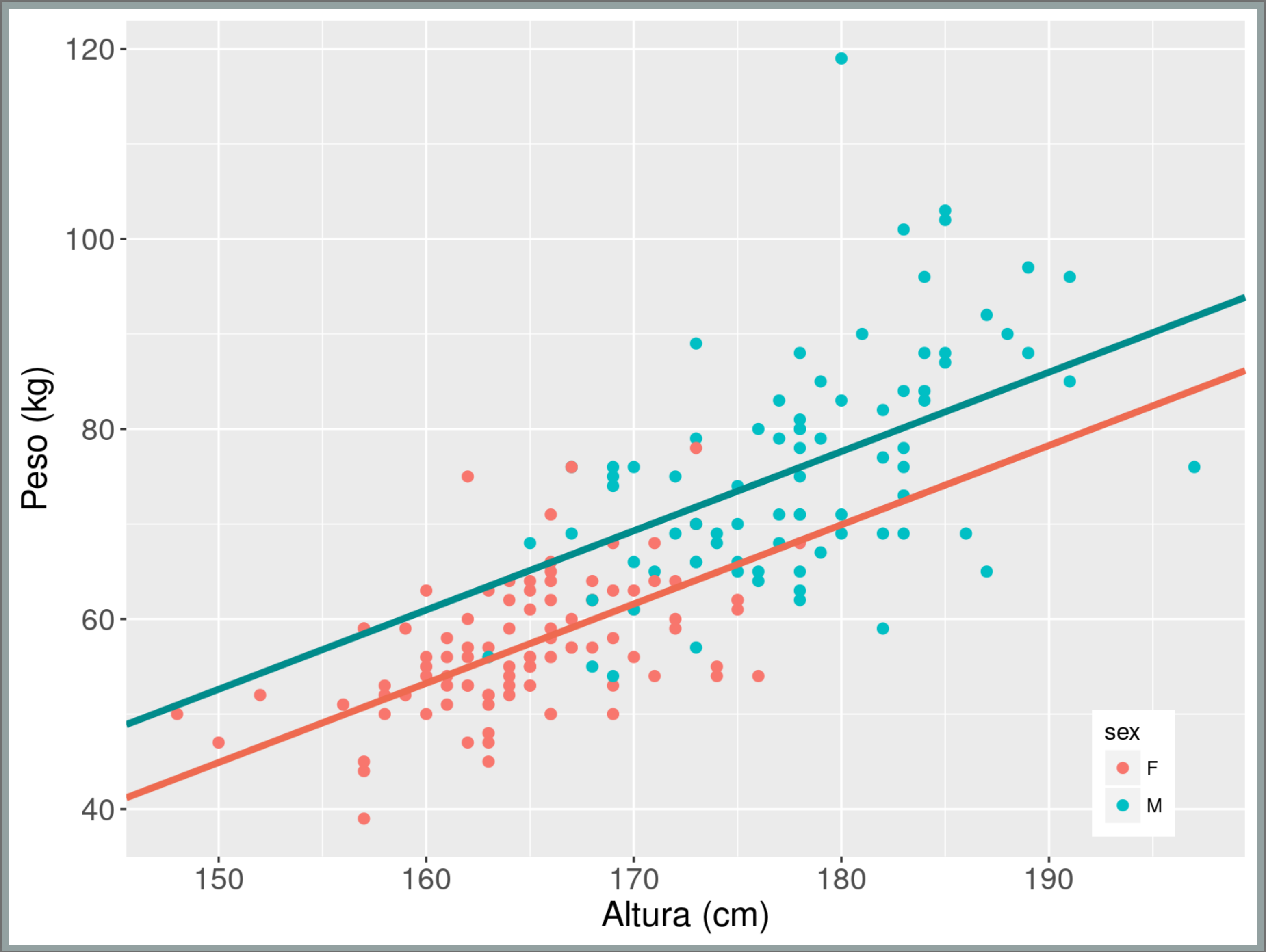
$$w_f = \hat{\alpha} + \hat{\beta}_h * height$$

Homem (sex=1)

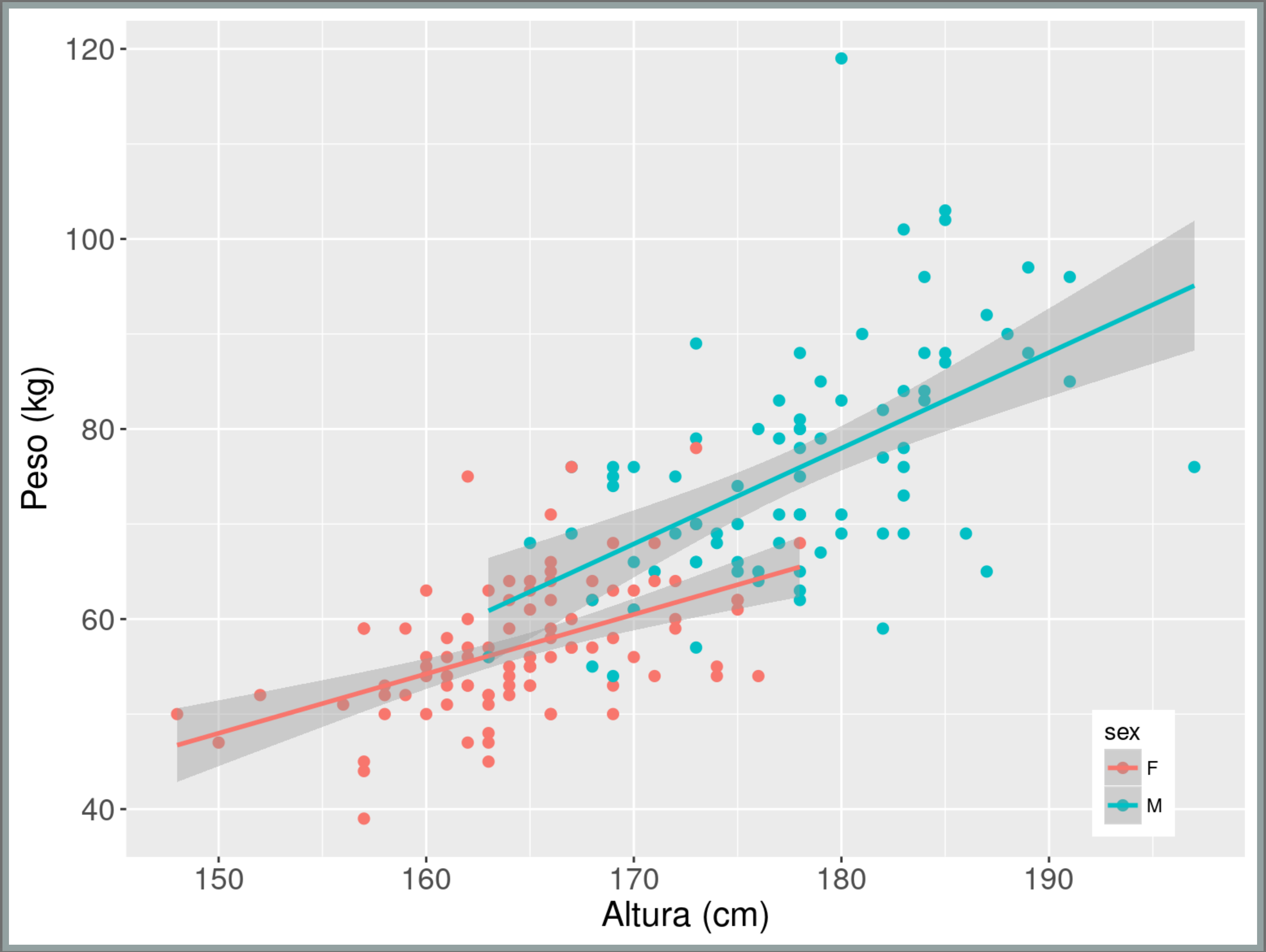
$$w_m = \hat{\alpha} + \hat{\beta}_s * sex + \hat{\beta} * height$$

$$w_m = \hat{\alpha} + \hat{\beta}_s + \hat{\beta}_h * height$$

# weight ~ height + sex



# weight ~ height + sex + height:sex



# weight ~ height + sex + height:sex

```
##
## Call:
## lm(formula = weight ~ height + sex + sex:height
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -20.990  -4.548  -0.926   4.821  41.023
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|
## (Intercept) -45.7988    24.8453  -1.843   0.067
## height        0.6252     0.1507   4.148 5.22e-0
## sexM        -57.4326    34.8293  -1.649   0.100
## height:sexM   0.3815     0.2037   1.873   0.062
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
##
## Residual standard error: 8.2 on 176 degrees of 
## Multiple R-squared:  0.6344, Adjusted R-squared
## F-statistic: 101.8 on 3 and 176 DF,  p-value: <
```

# lm(weight ~ height + sex + sex:height, data=Davis)

```
## (Intercept)      height        sexM height:sexM
## -45.7988220   0.6252035 -57.4326307   0.3815088
```

Feminino (sex = 0)

$$w = \hat{\alpha} + \hat{\beta}_s sex + \hat{\beta}_h height + \hat{\beta}_{s:h} sex * height$$
$$w_m = \hat{\alpha} + \hat{\beta}_h height$$

Masculinos (sex = 1)

$$w = \hat{\alpha} + \hat{\beta}_s sex + \hat{\beta}_h height + \hat{\beta}_{h:s} sex * height$$
$$w_h = \hat{\alpha} + \hat{\beta}_s + (\hat{\beta}_h + \hat{\beta}_{h:s}) * height$$

# Predição do modelo

Uma mulher de 161 cm de altura

$$w = \hat{\alpha} + \hat{\beta}_s sex + \hat{\beta}_h height + \hat{\beta}_{s:h} sex * height$$

$$sex = 0$$

```
## (Intercept)        height         sexM height:sexM
## -45.7988220    0.6252035 -57.4326307    0.3815088
```

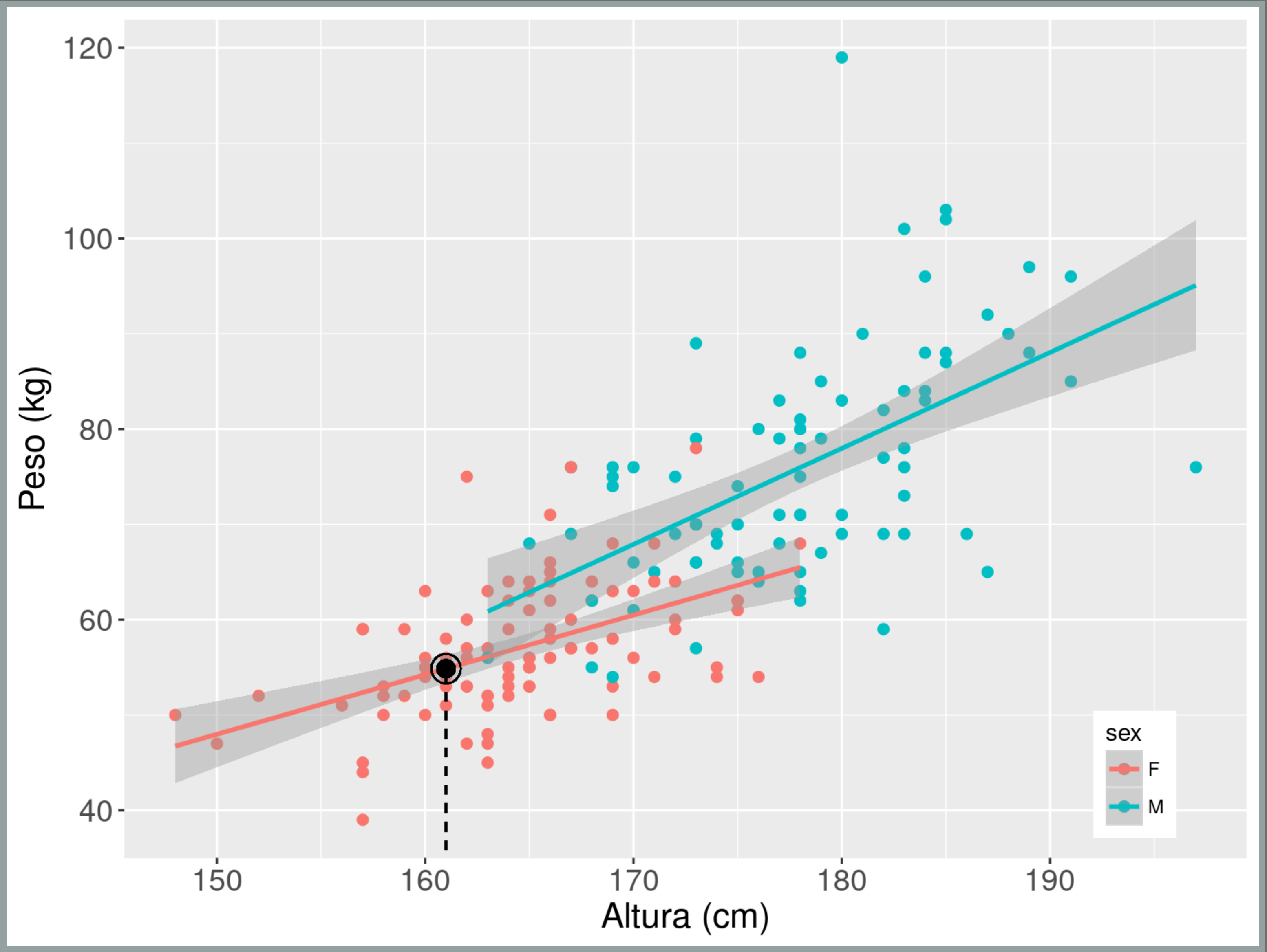
$$w = \hat{\alpha} + \hat{\beta}_h height$$

```
-45.8 + 0.625 * 161
```

```
## (Intercept)
##    54.85893
```

# Predito pelo modelo

- Uma mulher com 161cm de altura tem peso 54.86 kg.



120
100
80
60
40
Peso (kg)
150
160
170
180
190
Altura (cm)
sex
F
M

# Predito do Modelo

## Homem com 182 cm

$$w = \hat{\alpha} + \hat{\beta}_s sex + \hat{\beta}_h height + \hat{\beta}_{s:h} sex * height$$
$$sex = 1$$

```
## (Intercept)       height          sexM height:sexM
## -45.7988220    0.6252035 -57.4326307    0.3815088
```

$$w = \hat{\alpha} + \hat{\beta}_s + \hat{\beta}_h height + \hat{\beta}_{s:h} * height$$

$$w = \hat{\alpha} + \hat{\beta}_s + (\hat{\beta}_h + \hat{\beta}_{s:h}) * height$$

```
-45.8 -57.4 + 0.625 * 182 + 0.381 * 182
```

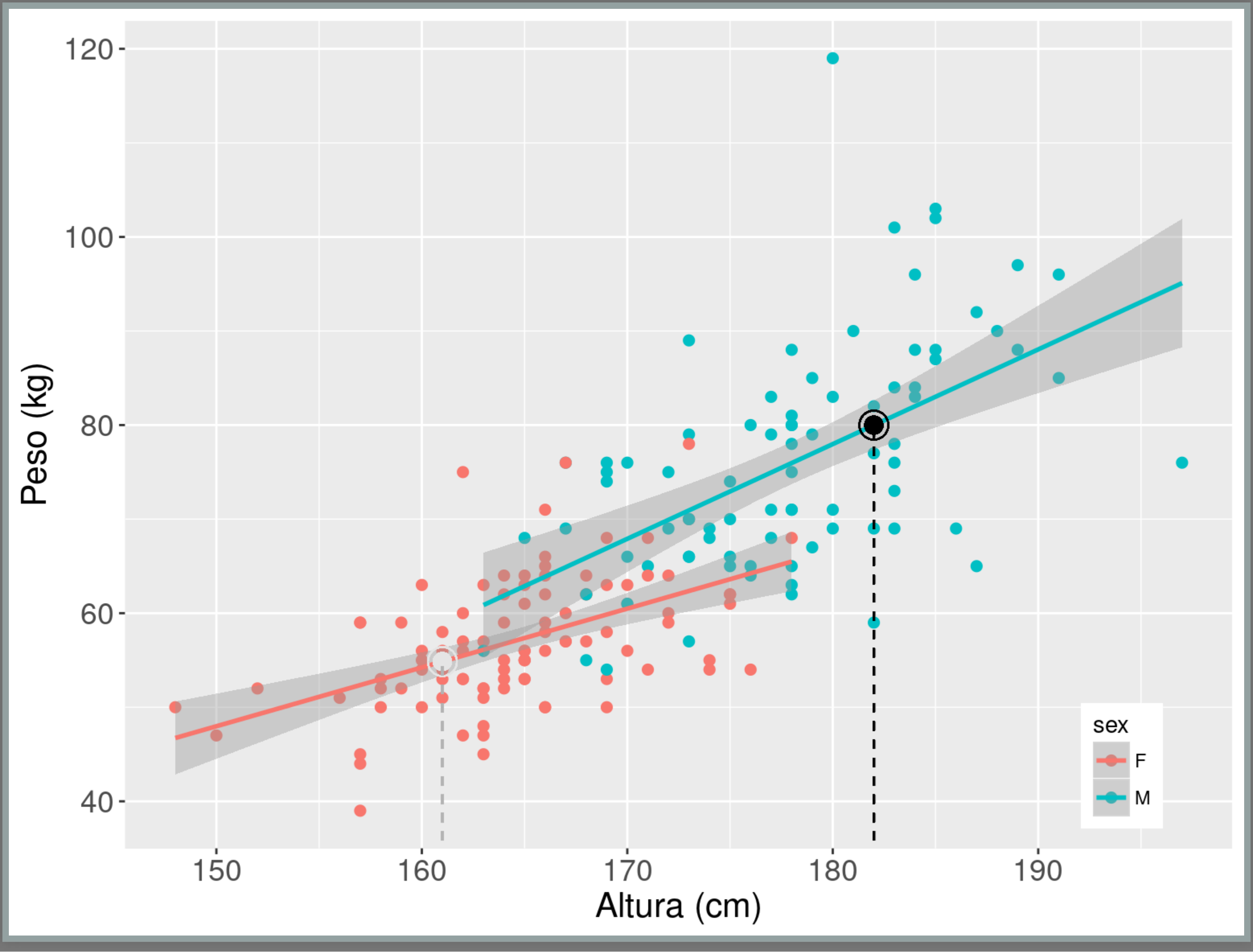
```
predHomem
```

```
## (Intercept)
##    79.99018
```

```
## 79.99018
```

# lm(weight ~ height + sex + sex:height, data=Davis)

- Um homem com 182cm de altura tem peso 79.99 kg.



# Matrix do Modelo

Primeiros registros nos dados

```
##   sex weight height
## 1   M     77    182
## 2   F     58    161
```

Matrix do Modelo (linhas 1 e 2)

```
##   (Intercept) height sexM height:sexM
## 1           1    182    1         182
## 2           1    161    0           0
```

Coeficientes do Modelo

```
## (Intercept)      height        sexM height:sexM
## -45.7988220   0.6252035 -57.4326307   0.3815088
```

Multiplicação Matricial

```
model.matrix(lmdavisfull)[1:2,] %*% coef(lmdavisfull)
```

```
##       [,1]
## 1 79.99018
## 2 54.85893
```

# Qual o melhor modelo?

## Princípio da parcimônia (Navalha de Occam)

- mínimo número de parâmetros
- linear é melhor que não-linear
- reter menos pressupostos
- simplificado ao mínimo adequado
- explicações mais simples são preferíveis

# Simplificação do modelo

## Método do modelo cheio ao mínimo adequado

1. ajuste o modelo máximo (cheio)
2. simplifique o modelo:
   - inspecione os coeficientes (summary)
   - remova termos não significativos
3. ordem de remoção de termos:
   - interação não significativos (maior ordem)
   - termos quadráticos ou não lineares
   - variáveis explicativas não significativas
   - agrupe níveis de fatores sem diferença
   - ANCOVA: intercepto não significativo -> 0

# Simplificação do modelo: continuação

## Compare o modelo anterior com o simplificado

A diferença não é significativa:

```
* retenha o modelo mais simples
* continue simplificando
```

A difereça é significativa

```
* retenha o modelo complexo
* modelo MINÍMO ADEQUADO
```

# Simplificando Modelo: exemplo

## Modelo cheio

```
##
## Call:
## lm(formula = weight ~ height + sex + sex:height
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -20.990  -4.548  -0.926   4.821  41.023
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|
## (Intercept) -45.7988    24.8453  -1.843   0.067
## height        0.6252     0.1507   4.148 5.22e-0
## sexM        -57.4326    34.8293  -1.649   0.100
## height:sexM   0.3815     0.2037   1.873   0.062
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
##
## Residual standard error: 8.2 on 176 degrees of
## Multiple R-squared:  0.6344, Adjusted R-squared
## F-statistic: 101.8 on 3 and 176 DF,  p-value: <
```

# Simplificando Modelo: exemplo

weight ~ height + sex + sex:height

weight ~ height + sex

```r
anova(lmdavisfull, lmdavis01)
```

```
## Analysis of Variance Table
##
## Model 1: weight ~ height + sex + sex:height
## Model 2: weight ~ height + sex
##   Res.Df   RSS Df Sum of Sq      F  Pr(>F)
## 1    176 11833
## 2    177 12069 -1   -235.82 3.5075 0.06275 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
```

# Simplificando Modelo: exemplo

weight ~ height + sex

```
##
## Call:
## lm(formula = weight ~ height + sex, data = Davi
##
## Residuals:
##     Min       1Q   Median       3Q      Max
## -20.302  -4.808  -0.335   5.239  41.366
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|
## (Intercept) -80.2107    16.8415  -4.763 3.96e-0
## height        0.8341     0.1021   8.169 5.71e-1
## sexM          7.7070     1.8345   4.201 4.20e-0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
##
## Residual standard error: 8.258 on 177 degrees o
## Multiple R-squared:  0.6271, Adjusted R-squared
## F-statistic: 148.8 on 2 and 177 DF,  p-value: <
```

# Simplificando Modelo: exemplo

weight ~ height + sex

weight ~ height

```
## Analysis of Variance Table
##
## Model 1: weight ~ height + sex
## Model 2: weight ~ height
##   Res.Df   RSS Df Sum of Sq     F    Pr(>F)
## 1    177 12069
## 2    178 13273 -1   -1203.5 17.65 4.204e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
```

# Modelo Mínimo Adequado

```
summary(lmdavis01)
```

```
##
## Call:
## lm(formula = weight ~ height + sex, data = Davi
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -20.302  -4.808  -0.335   5.239  41.366
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|
## (Intercept) -80.2107    16.8415  -4.763 3.96e-0
## height        0.8341     0.1021   8.169 5.71e-1
## sexM          7.7070     1.8345   4.201 4.20e-0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
##
## Residual standard error: 8.258 on 177 degrees o
## Multiple R-squared:  0.6271, Adjusted R-squared
## F-statistic: 148.8 on 2 and 177 DF,  p-value: <
```

# Modelo Mínimo Adequado

```r
anova(lmdavis01)
```

```
## Analysis of Variance Table
##
## Response: weight
##            Df  Sum Sq Mean Sq F value    Pr(>F)
## height      1 19095.0 19095.0  280.04 < 2.2e-16
## sex         1  1203.5  1203.5   17.65 4.204e-05
## Residuals 177 12069.2    68.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
```

# Modelo Mínimo Adequado

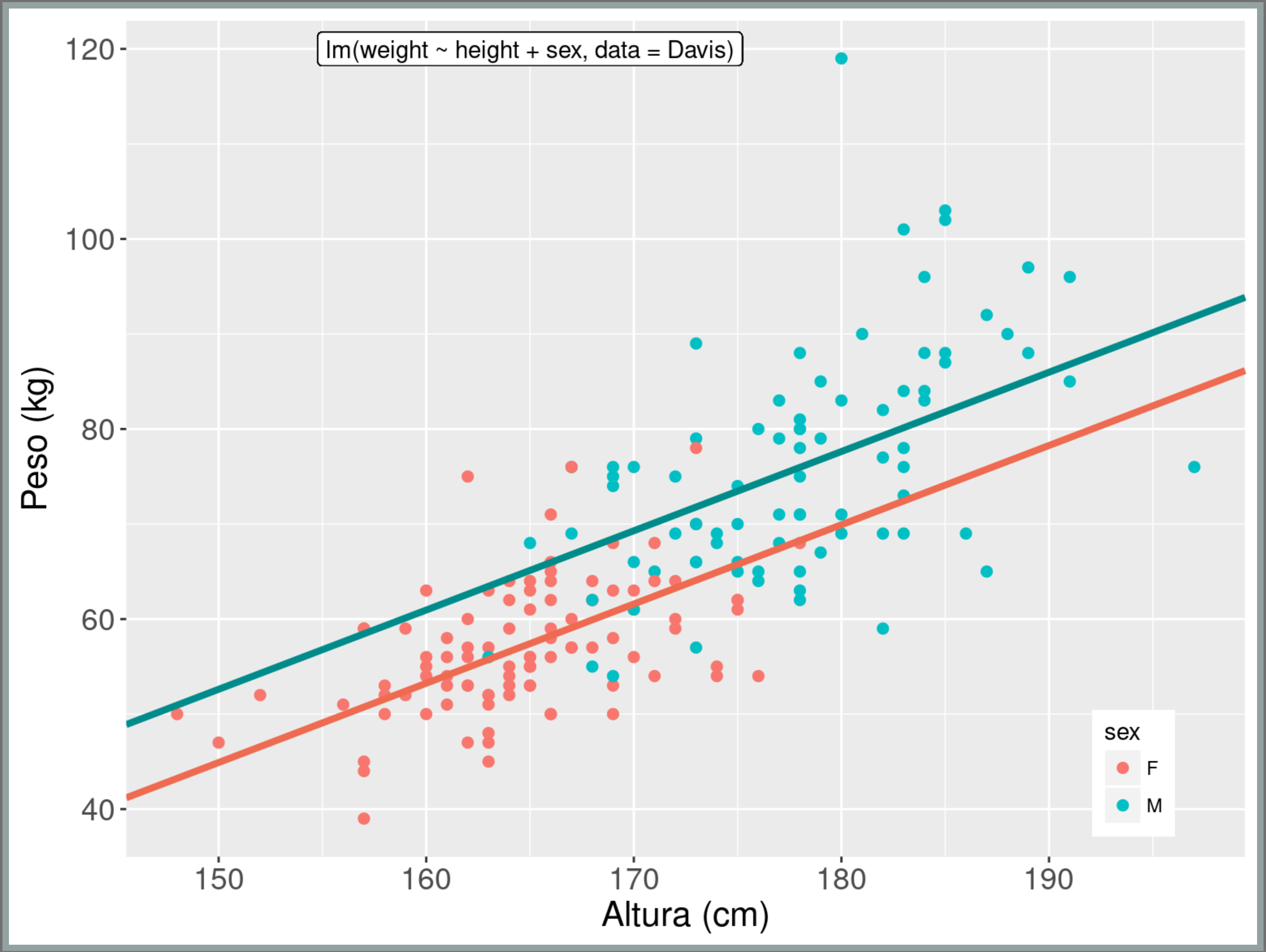
```r
coef(lmdavis01)
```

```
## (Intercept)      height        sexM
## -80.2107328   0.8340964   7.7070166
```

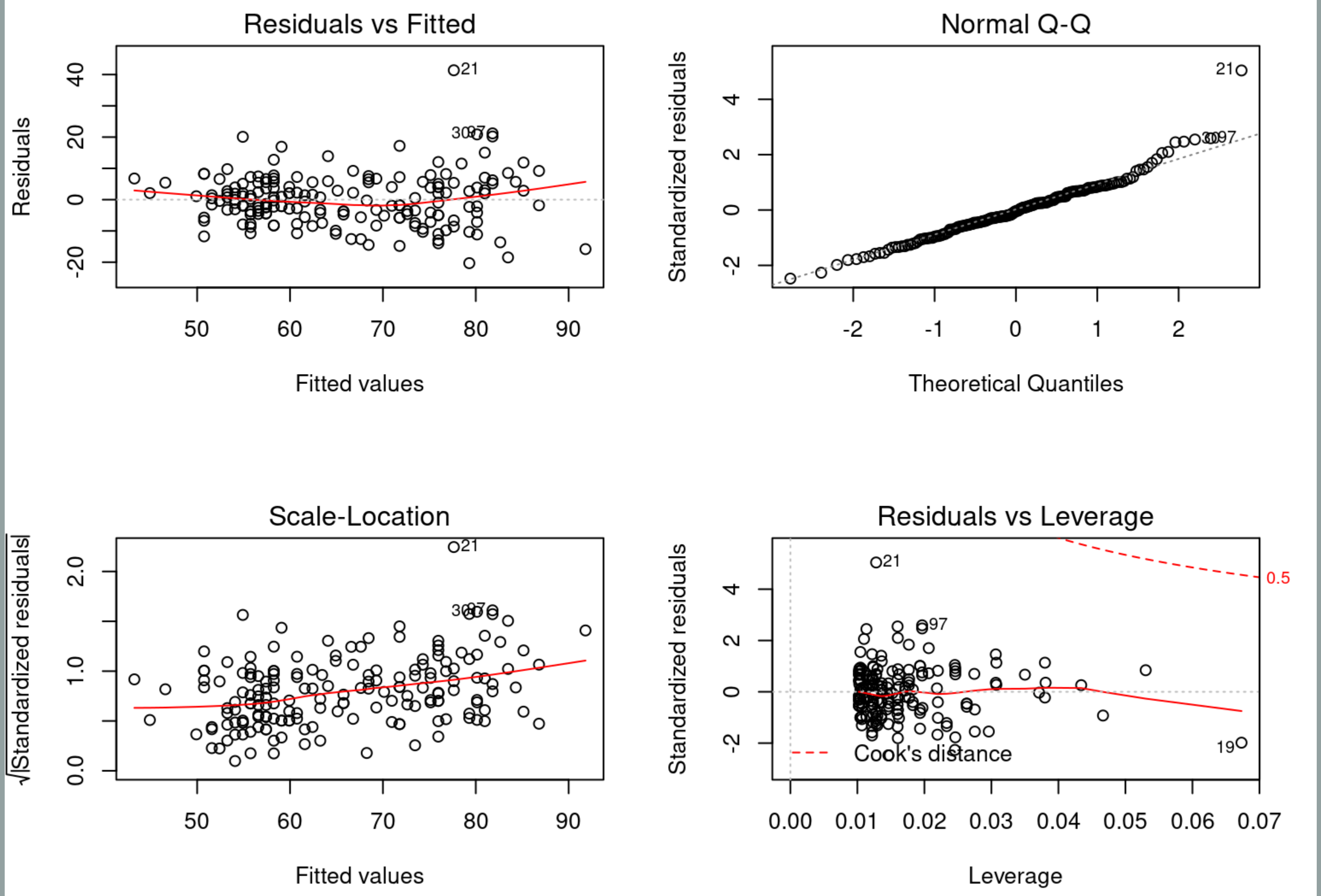
```r
confint(lmdavis01)
```

```
##                  2.5 %     97.5 %
## (Intercept) -113.44661 -46.974852
## height         0.63259   1.035603
## sexM           4.08671  11.327323
```

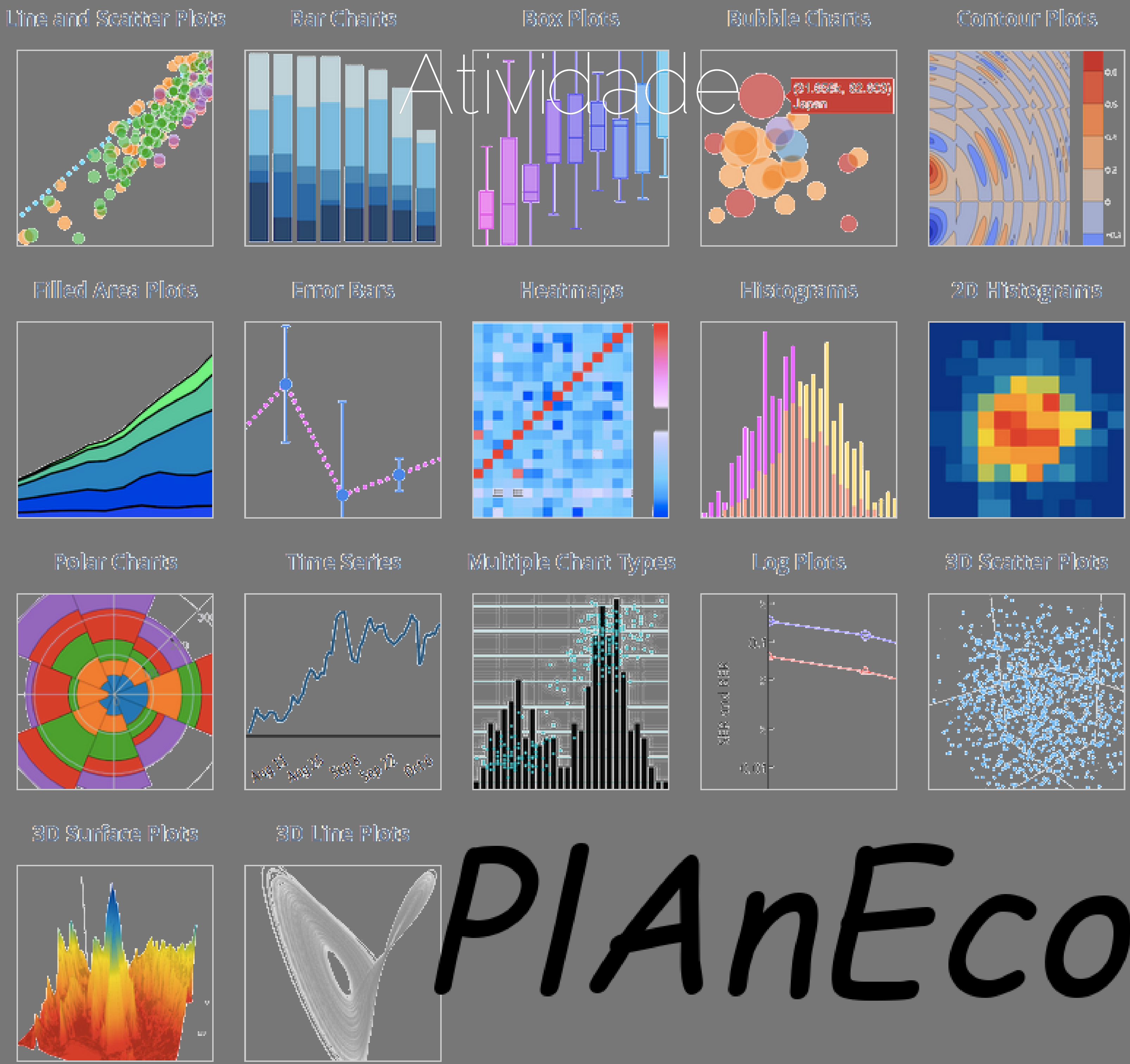
# Modelo Mínimo Adequado



# Diagnóstico do Modelo:



Atividade

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# Poluição

# Modelo Linear Múltiplo:

- quais variáveis incluir
- curvatura em resposta a variável preditora
- interações entre variáveis
- correlação entre variáveis preditoras (colinearidade)
- saturação do modelo (complexidade

# Poluição: ozônio

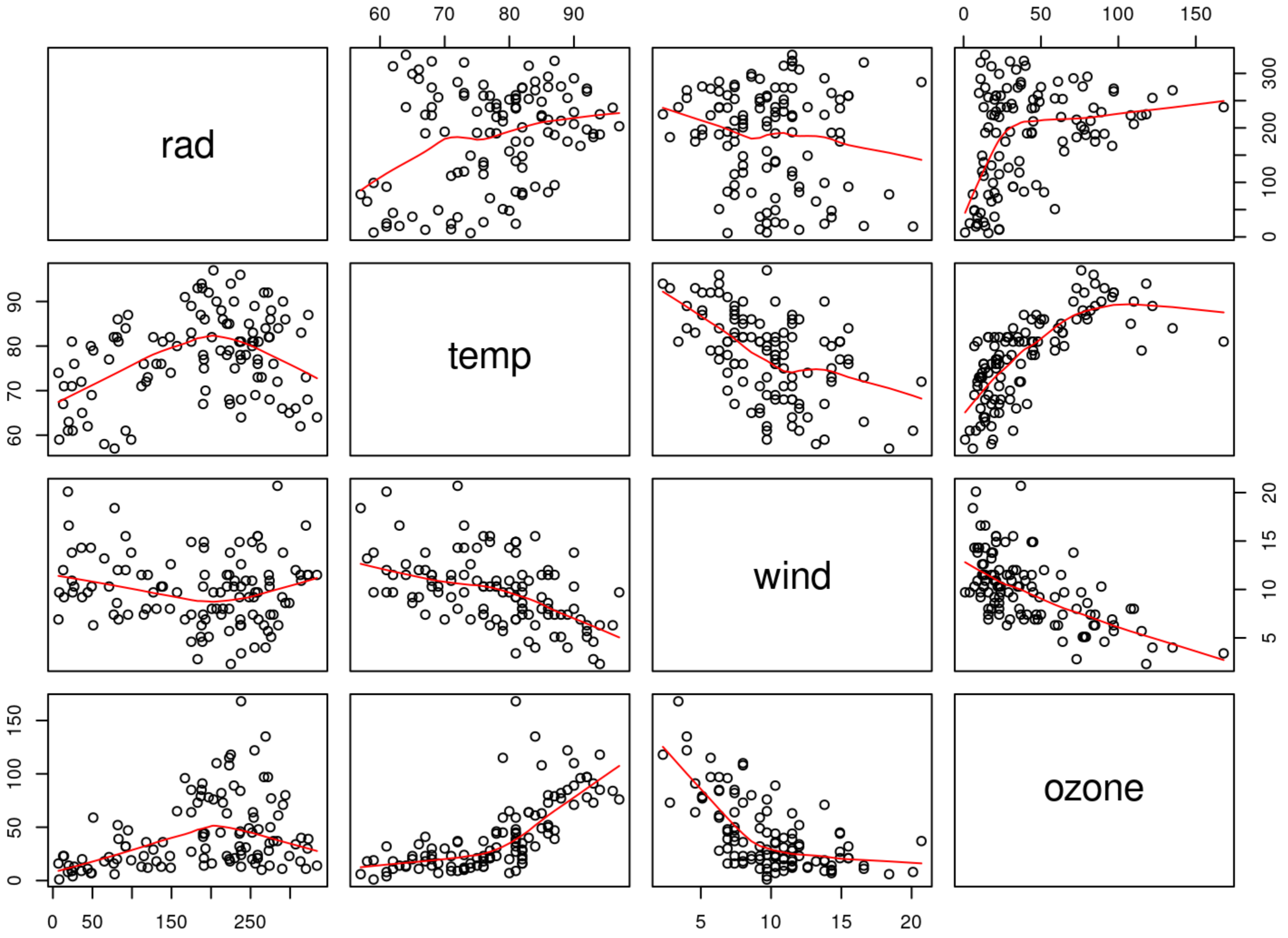
Quais variáveis climáticas estão relacionadas à concentração de ozônio?

| rad | temp | wind | ozone |
|---|---|---|---|
| 190 | 67 | 7.4 | 41 |
| 118 | 72 | 8.0 | 36 |
| 149 | 74 | 12.6 | 12 |
| 313 | 62 | 11.5 | 18 |
| 299 | 65 | 8.6 | 23 |
| 99 | 59 | 13.8 | 19 |
| 19 | 61 | 20.1 | 8 |
| 256 | 69 | 9.7 | 16 |
| 290 | 66 | 9.2 | 11 |
| 274 | 68 | 10.9 | 14 |

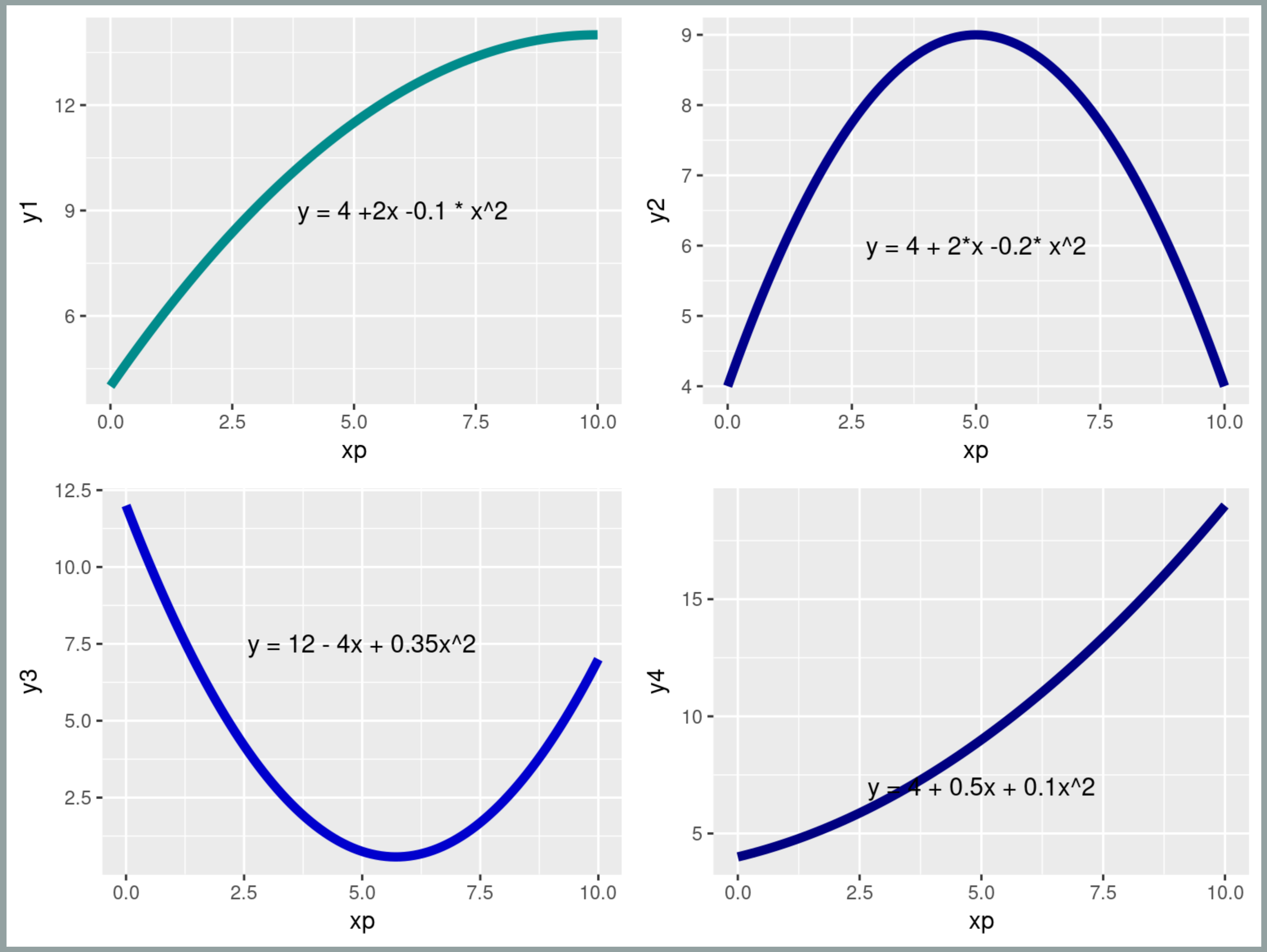
# Ozônio data

| var | natureza | tipo | descrição |
|---|---|---|---|
| rad | pred | contínua | radiação |
| temp | pred | contínua | temperatura |
| wind | pred | contínua | vento |
| ozone | resposta | contínua | ozônio |

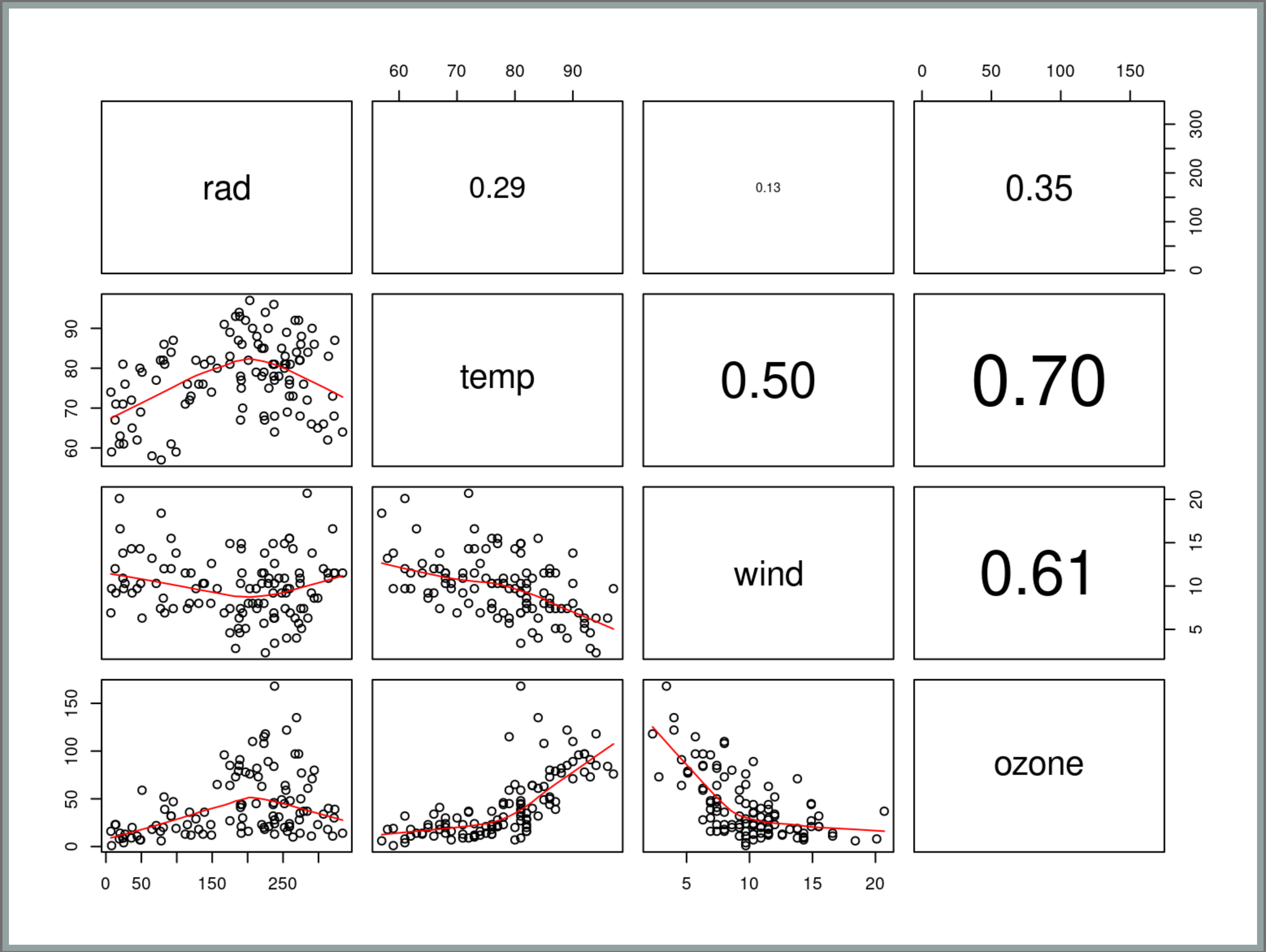
# Linearidade



# Curvatura da relação: polinômios



# Correlação entre preditoras



# Correlação entre preditoras

## Indíce de colinearidade (confirmar)

VIF: Variance Inflation Factor

Proporcional a variação compartilhada com outras preditoras

$$VIF = \frac{1}{1 - R_k^2}$$

$R_k^2$ : coeficiente de determinação da preditora (k) em relação a outras preditoras do modelo

- $VIF = 1$ : não há variação compartilhada;
- $VIF = 4$ : 75% de variação explicada ;
- $VIF = 10$ : 90% de variação explicada;

# Colinearidade: soluções

- reter apenas uma das variáveis colineares
- reduzir as dimensões das variáveis colineares (PCA)

# Definir os termos do modelo cheio

## Modelo para concentração de ozônio:

Modelo Cheio:

- temp
- wind
- rad
- temp^2
- wind^2
- rad^2
- temp : wind
- temp : rad
- wind : rad
- temp : wind : rad

# Modelo Cheio: Ozônio

```
lmozfull <- lm(ozone ~ temp + wind + rad +
                  I(temp^2)+ I(wind^2) + I(rad^2) +
                 temp:wind + temp:rad + wind:rad +
                 temp:wind:rad, data = ozo)
lmozfull <- lm(ozone ~  temp * wind * rad +
                  I(temp^2) + I(wind^2) + I(rad^2), data = ozo)
```

```
##
## Call:
## lm(formula = ozone ~ temp * wind * rad + I(temp^2) + I(wind^2) +
##     I(rad^2), data = ozo)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -38.894 -11.205  -2.736   8.809  70.551
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)    5.683e+02  2.073e+02   2.741  0.00725 **
## temp          -1.076e+01  4.303e+00  -2.501  0.01401 *
## wind          -3.237e+01  1.173e+01  -2.760  0.00687 **
## rad           -3.117e-01  5.585e-01  -0.558  0.57799
## I(temp^2)      5.833e-02  2.396e-02   2.435  0.01668 *
## I(wind^2)      6.106e-01  1.469e-01   4.157 6.81e-05 ***
## I(rad^2)      -3.619e-04  2.573e-04  -1.407  0.16265
## temp:wind      2.377e-01  1.367e-01   1.739  0.08519 .
## temp:rad       8.403e-03  7.512e-03   1.119  0.26602
## wind:rad       2.054e-02  4.892e-02   0.420  0.67552
## temp:wind:rad -4.324e-04  6.595e-04  -0.656  0.51358
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17.82 on 100 degrees of freedom
## Multiple R-squared:  0.7394, Adjusted R-squared:  0.7133
## F-statistic: 28.37 on 10 and 100 DF,  p-value: < 2.2e-16
```

# Simplificando o modelo: Ozônio

```r
lmoz01 <- lm(ozone ~ temp + wind + rad +
                  I(temp^2)+ I(wind^2) + I(rad^2) +
                 temp:wind + temp:rad + wind:rad, data = ozo)
anova(lmozfull, lmoz01)
```

```
## Analysis of Variance Table
##
## Model 1: ozone ~ temp * wind * rad + I(temp^2) + I(wind^2) + I(:
## Model 2: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) + I(:
##     temp:wind + temp:rad + wind:rad
##   Res.Df   RSS Df Sum of Sq      F Pr(>F)
## 1    100 31742
## 2    101 31879 -1   -136.44 0.4298 0.5136
```

```
##
## Call:
## lm(formula = ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) +
##     I(rad^2) + temp:wind + temp:rad + wind:rad, data = ozo)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -39.611 -11.455  -2.901   8.548  70.325
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.245e+02  1.957e+02   2.680   0.0086 **
## temp        -1.021e+01  4.209e+00  -2.427   0.0170 *
## wind        -2.802e+01  9.645e+00  -2.906   0.0045 **
## rad          2.628e-02  2.142e-01   0.123   0.9026
## I(temp^2)    5.953e-02  2.382e-02   2.499   0.0141 *
## I(wind^2)    6.173e-01  1.461e-01   4.225 5.25e-05 ***
## I(rad^2)    -3.388e-04  2.541e-04  -1.333   0.1855
## temp:wind    1.734e-01  9.497e-02   1.825   0.0709 .
## temp:rad     3.750e-03  2.459e-03   1.525   0.1303
## wind:rad    -1.127e-02  6.277e-03  -1.795   0.0756 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17.77 on 101 degrees of freedom
## Multiple R-squared:  0.7383, Adjusted R-squared:  0.715
## F-statistic: 31.66 on 9 and 101 DF,  p-value: < 2.2e-16
```

# Simplificando o modelo: Ozônio

```r
lmoz02 <- lm(ozone ~ temp + wind + rad +
                  I(temp^2)+ I(wind^2) + I(rad^2) +
                 temp:wind +  wind:rad, data = ozo)
anova(lmoz01, lmoz02)
```

```
## Analysis of Variance Table
##
## Model 1: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) + I(:
##     temp:wind + temp:rad + wind:rad
## Model 2: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) + I(:
##     temp:wind + wind:rad
##   Res.Df   RSS Df Sum of Sq      F Pr(>F)
## 1    101 31879
## 2    102 32613 -1   -734.23 2.3262 0.1303
```

```
##
## Call:
## lm(formula = ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) +
##     I(rad^2) + temp:wind + wind:rad, data = ozo)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -42.040 -11.962  -2.863   9.661  70.475
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.488e+02  1.963e+02   2.796  0.00619 **
## temp        -1.144e+01  4.158e+00  -2.752  0.00702 **
## wind        -2.876e+01  9.695e+00  -2.967  0.00375 **
## rad          3.061e-01  1.113e-01   2.751  0.00704 **
## I(temp^2)    7.145e-02  2.265e-02   3.154  0.00211 **
## I(wind^2)    6.363e-01  1.465e-01   4.343 3.33e-05 ***
## I(rad^2)    -2.690e-04  2.516e-04  -1.069  0.28755
## temp:wind    1.840e-01  9.533e-02   1.930  0.05644 .
## wind:rad    -1.381e-02  6.090e-03  -2.268  0.02541 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17.88 on 102 degrees of freedom
## Multiple R-squared:  0.7322, Adjusted R-squared:  0.7112
## F-statistic: 34.87 on 8 and 102 DF,  p-value: < 2.2e-16
```

# Simplificando o modelo: Ozônio

```
lmoz03 <- lm(ozone ~ temp + wind + rad +
                  I(temp^2)+ I(wind^2) +
                  temp:wind +  wind:rad, data = ozo)
anova( lmoz02, lmoz03)
```

```
## Analysis of Variance Table
##
## Model 1: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) + I(:
##     temp:wind + wind:rad
## Model 2: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) + ter
##     wind:rad
##   Res.Df   RSS Df Sum of Sq     F Pr(>F)
## 1    102 32613
## 2    103 32978 -1   -365.45 1.143 0.2875
```

```
##
## Call:
## lm(formula = ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) +
##     temp:wind + wind:rad, data = ozo)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -41.379 -11.375  -2.217   8.921  71.247
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 514.401470 193.783580   2.655  0.00920 **
## temp        -10.654041   4.094889  -2.602  0.01064 *
## wind        -27.391965   9.616998  -2.848  0.00531 **
## rad           0.212945   0.069283   3.074  0.00271 **
## I(temp^2)     0.067805   0.022408   3.026  0.00313 **
## I(wind^2)     0.619396   0.145773   4.249 4.72e-05 ***
## temp:wind     0.169674   0.094458   1.796  0.07538 .
## wind:rad     -0.013561   0.006089  -2.227  0.02813 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17.89 on 103 degrees of freedom
## Multiple R-squared:  0.7292, Adjusted R-squared:  0.7108
## F-statistic: 39.63 on 7 and 103 DF,  p-value: < 2.2e-16
```

# Simplificando o modelo: Ozônio

```r
lmoz04 <- lm(ozone ~ temp + wind + rad +
                 I(temp^2)+ I(wind^2) +
                 wind:rad, data = ozo)
anova( lmoz03, lmoz04)
```

```
## Analysis of Variance Table
##
## Model 1: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) + ter
##     wind:rad
## Model 2: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) + wi
##   Res.Df   RSS Df Sum of Sq      F  Pr(>F)
## 1    103 32978
## 2    104 34011 -1   -1033.1 3.2267 0.07538 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Call:
## lm(formula = ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) +
##     wind:rad, data = ozo)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -44.478 -10.735  -2.437   9.685  77.543
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 223.573855 107.618223   2.077 0.040221 *
## temp         -5.197139   2.775039  -1.873 0.063902 .
## wind        -10.816032   2.736757  -3.952 0.000141 ***
## rad           0.173431   0.066398   2.612 0.010333 *
## I(temp^2)     0.043640   0.018112   2.410 0.017731 *
## I(wind^2)     0.430059   0.101767   4.226 5.12e-05 ***
## wind:rad     -0.009819   0.005783  -1.698 0.092507 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.08 on 104 degrees of freedom
## Multiple R-squared:  0.7208, Adjusted R-squared:  0.7047
## F-statistic: 44.74 on 6 and 104 DF,  p-value: < 2.2e-16
```

# Simplificando o modelo: Ozônio

```
lmoz05 <- lm(ozone ~ temp + wind + rad +
                  I(temp^2)+ I(wind^2),
                  data = ozo)
anova( lmoz04, lmoz05)
```

```
## Analysis of Variance Table
##
## Model 1: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2) + wi
## Model 2: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2)
##   Res.Df   RSS Df Sum of Sq     F  Pr(>F)
## 1    104 34011
## 2    105 34954 -1   -942.85 2.883 0.09251 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

# Simplificando o Modelo

ozone ~ temp + wind + rad + temp^2 + wind^2

```
##
## Call:
## lm(formula = ozone ~ temp + wind + rad + I(temp^2) + I(wind^2),
##     data = ozo)
##
## Residuals:
##     Min       1Q  Median      3Q     Max
## -48.044 -10.796  -4.138   8.131  80.098
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 291.16758  100.87723   2.886  0.00473 **
## temp         -6.33955    2.71627  -2.334  0.02150 *
## wind        -13.39674    2.29623  -5.834 6.05e-08 ***
## rad           0.06586    0.02005   3.285  0.00139 **
## I(temp^2)     0.05102    0.01774   2.876  0.00488 **
## I(wind^2)     0.46464    0.10060   4.619 1.10e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.25 on 105 degrees of freedom
## Multiple R-squared:  0.713,  Adjusted R-squared:  0.6994
## F-statistic: 52.18 on 5 and 105 DF,  p-value: < 2.2e-16
```

# Simplificando o modelo: Ozônio

```
lmoz06 <- lm(ozone ~ temp + wind + rad +
                 I(wind^2)
                 , data = ozo)
anova(lmoz05, lmoz06)
```

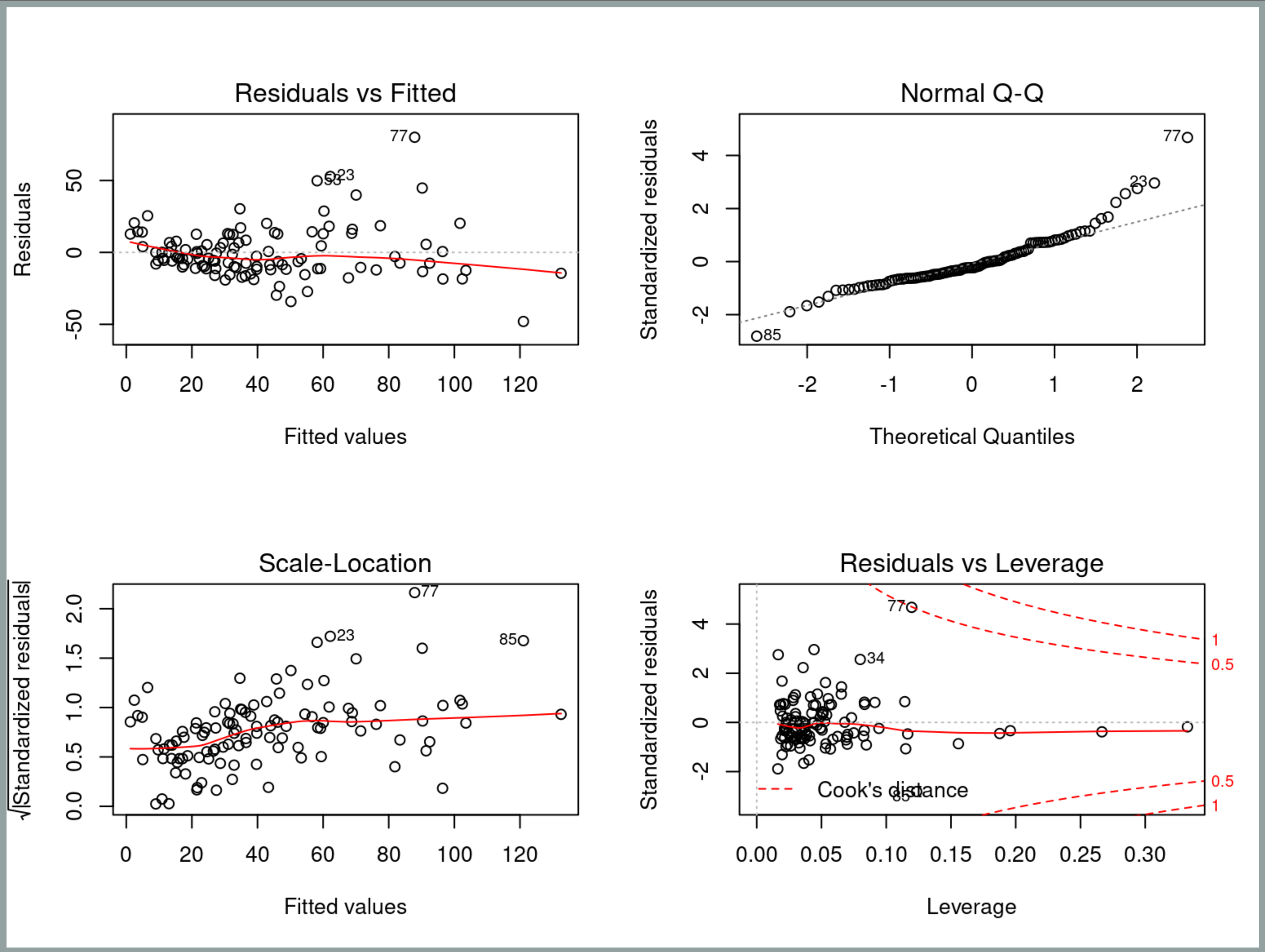
```
## Analysis of Variance Table
##
## Model 1: ozone ~ temp + wind + rad + I(temp^2) + I(wind^2)
## Model 2: ozone ~ temp + wind + rad + I(wind^2)
##   Res.Df   RSS Df Sum of Sq      F   Pr(>F)
## 1    105 34954
## 2    106 37708 -1   -2753.7 8.2718 0.004877 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

# Modelo Mínimo Adequado

ozone ~ temp + wind + rad + temp^2 + wind^2

```
##
## Call:
## lm(formula = ozone ~ temp + wind + rad + I(temp^2) + I(wind^2),
##     data = ozo)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -48.044 -10.796  -4.138   8.131  80.098
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 291.16758  100.87723   2.886  0.00473 **
## temp         -6.33955    2.71627  -2.334  0.02150 *
## wind        -13.39674    2.29623  -5.834 6.05e-08 ***
## rad           0.06586    0.02005   3.285  0.00139 **
## I(temp^2)     0.05102    0.01774   2.876  0.00488 **
## I(wind^2)     0.46464    0.10060   4.619 1.10e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.25 on 105 degrees of freedom
## Multiple R-squared:  0.713,  Adjusted R-squared:  0.6994
## F-statistic: 52.18 on 5 and 105 DF,  p-value: < 2.2e-16
```

# DIAGNÓSTICO DO MODELO



# Transformando variável

```r
lmoz07 <- lm(log(ozone) ~ temp + wind + rad +
    I(temp^2)+ I(wind^2) + I(rad^2) + temp:wind +
      temp:rad + wind:rad + temp:wind:rad, data =
 ozo)
summary(lmoz07)
```

```
##
## Call:
## lm(formula = log(ozone) ~ temp + wind + rad + I
##     I(rad^2) + temp:wind + temp:rad + wind:rad
##     data = ozo)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.91943 -0.24169 -0.01742  0.28213  1.11802
##
## Coefficients:
##                 Estimate Std. Error t value Pr(
## (Intercept)    2.803e+00  5.676e+00   0.494   0
## temp          -3.018e-02  1.178e-01  -0.256   0
## wind          -9.812e-02  3.211e-01  -0.306   0
## rad            2.771e-02  1.529e-02   1.812   0
```

```
## I(temp^2)       6.034e-04  6.559e-04   0.920    0
## I(wind^2)       8.732e-03  4.021e-03   2.172    0
## I(rad^2)       -1.489e-05  7.043e-06  -2.114    0
## temp:wind      -1.985e-03  3.742e-03  -0.530    0
## temp:rad       -2.507e-04  2.056e-04  -1.219    0
```

# Simplificando o modelo

```
lmoz08 <- lm(log(ozone) ~ temp + wind + rad +
                  I(temp^2)+ I(wind^2) + I(rad^2)
 +
                 temp:wind + temp:rad + wind:rad,
 data = ozo)
anova(lmoz07, lmoz08)
```

```
## Analysis of Variance Table
##
## Model 1: log(ozone) ~ temp + wind + rad + I(tem
##     temp:wind + temp:rad + wind:rad + temp:wind
## Model 2: log(ozone) ~ temp + wind + rad + I(tem
##     temp:wind + temp:rad + wind:rad
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1    100 23.787
## 2    101 24.256 -1  -0.46883 1.9709 0.1634
```

# Simplificando o modelo

```
 lmoz09 <- lm(log(ozone) ~ temp + wind + rad +
                 I(temp^2)+ I(wind^2) + I(rad^2)
  +
                temp:wind +  wind:rad, data =
  ozo)
anova(lmoz08, lmoz09)
```

```
## Analysis of Variance Table
##
## Model 1: log(ozone) ~ temp + wind + rad + I(tem
##     temp:wind + temp:rad + wind:rad
## Model 2: log(ozone) ~ temp + wind + rad + I(tem
##     temp:wind + wind:rad
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1    101 24.256
## 2    102 24.281 -1  -0.02515 0.1047 0.7469
```

# Simplificando o modelo

```
lmoz10 <- lm(log(ozone) ~ temp + wind + rad +
                 I(temp^2)+ I(wind^2) + I(rad^2)
 +
                 wind:rad, data = ozo)
anova(lmoz09, lmoz10)
```

```
## Analysis of Variance Table
##
## Model 1: log(ozone) ~ temp + wind + rad + I(tem
##     temp:wind + wind:rad
## Model 2: log(ozone) ~ temp + wind + rad + I(tem
##     wind:rad
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1    102 24.281
## 2    103 24.401 -1  -0.11987 0.5035 0.4796
```

# Simplificando o modelo

```
lmoz11 <- lm(log(ozone) ~ temp + wind + rad +
             I(temp^2)+ I(wind^2) + I(rad^2), data =
  ozo)
anova(lmoz10, lmoz11)
```

```
## Analysis of Variance Table
##
## Model 1: log(ozone) ~ temp + wind + rad + I(tem
##     wind:rad
## Model 2: log(ozone) ~ temp + wind + rad + I(tem
##   Res.Df    RSS Df Sum of Sq    F Pr(>F)
## 1    103 24.401
## 2    104 24.522 -1  -0.12081 0.51 0.4768
```

# Simplificando o modelo

```
lmoz12 <- lm(log(ozone) ~ temp + wind + rad +
            I(wind^2) + I(rad^2), data = ozo)
anova(lmoz11, lmoz12)
```

```
## Analysis of Variance Table
##
## Model 1: log(ozone) ~ temp + wind + rad + I(tem
## Model 2: log(ozone) ~ temp + wind + rad + I(win
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1    104 24.522
## 2    105 24.707 -1  -0.18512 0.7851 0.3776
```

# Simplificando o modelo

```
summary( lmoz12)
```

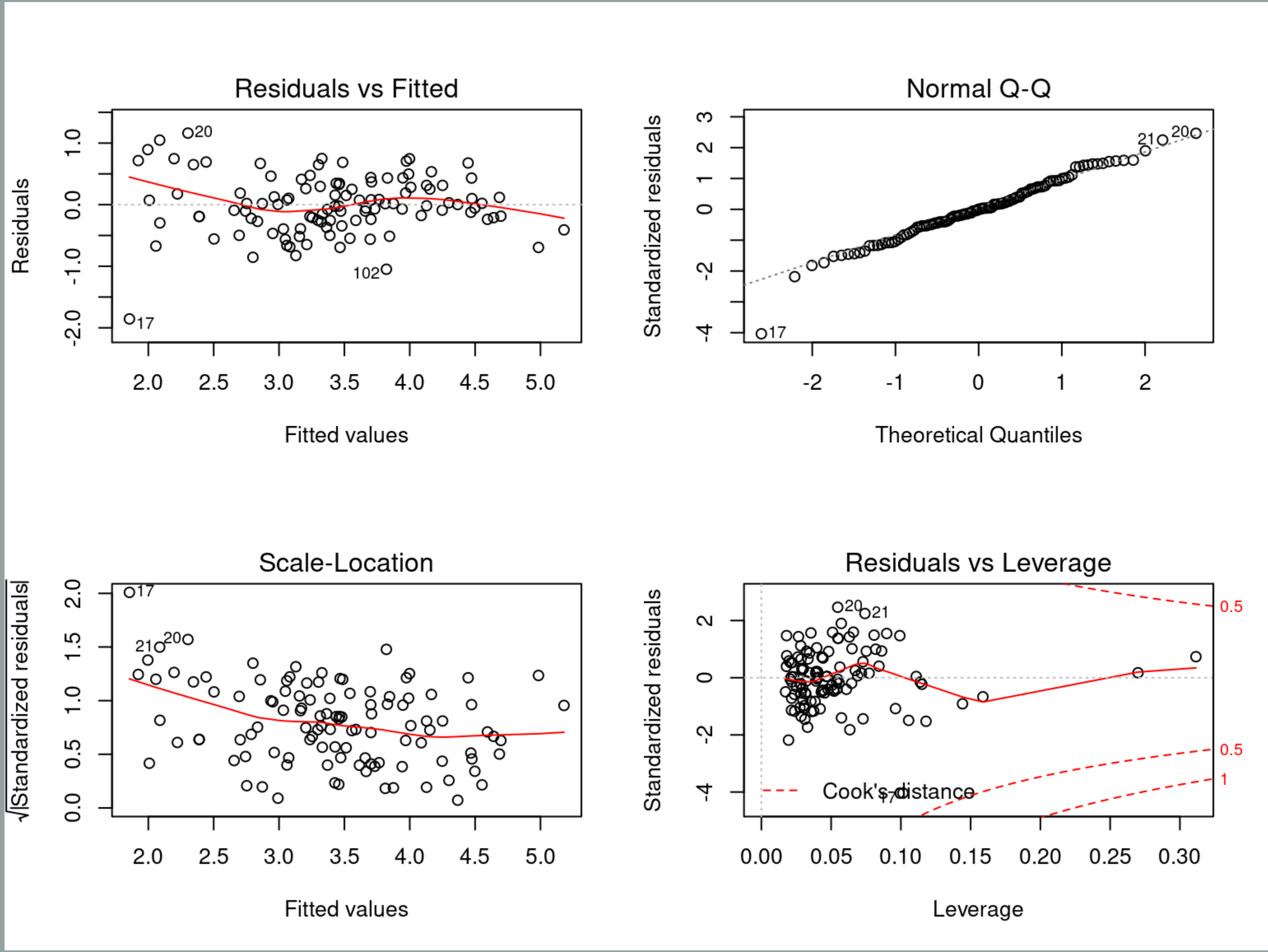
```
##
## Call:
## lm(formula = log(ozone) ~ temp + wind + rad + I
##     data = ozo)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.85551 -0.25578  0.00248  0.31349  1.16251
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|
## (Intercept)  7.724e-01  6.350e-01   1.216 0.226
## temp         4.193e-02  6.237e-03   6.723 9.52e
## wind        -2.211e-01  5.874e-02  -3.765 0.000
## rad          7.466e-03  2.323e-03   3.215 0.001
## I(wind^2)    7.390e-03  2.585e-03   2.859 0.005
## I(rad^2)    -1.470e-05  6.734e-06  -2.183 0.031
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
##
## Residual standard error: 0.4851 on 105 degrees
```

# Simplificando o modelo

```r
 lmoz13 <- lm(log(ozone) ~ temp + wind + rad +
          I(wind^2), data = ozo)
anova(lmoz12, lmoz13)
```

```
## Analysis of Variance Table
##
## Model 1: log(ozone) ~ temp + wind + rad + I(win
## Model 2: log(ozone) ~ temp + wind + rad + I(win
##   Res.Df    RSS Df Sum of Sq      F   Pr(>F)
## 1    105 24.707
## 2    106 25.828 -1   -1.1216 4.7665 0.03125 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
```

# Diagnóstico do Modelo



# Modelo Mínimo Adequado

```
summary(lmoz12)
```

```
##
## Call:
## lm(formula = log(ozone) ~ temp + wind + rad + I
##     data = ozo)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.85551 -0.25578  0.00248  0.31349  1.16251
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|
## (Intercept)  7.724e-01  6.350e-01   1.216 0.226
## temp         4.193e-02  6.237e-03   6.723 9.52e
## wind        -2.211e-01  5.874e-02  -3.765 0.000
## rad          7.466e-03  2.323e-03   3.215 0.001
## I(wind^2)    7.390e-03  2.585e-03   2.859 0.005
## I(rad^2)    -1.470e-05  6.734e-06  -2.183 0.031
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.0
##
## Residual standard error: 0.4851 on 105 degrees
```

# IMPORTÂNCIA DAS VARIÁVEIS

Escalas diferentes

```
lmoz12a <- lm(log(ozone) ~ I(temp/100) + wind + rad +
          I((wind/100)^2) + I(rad^2), data = ozo)
anova(lmoz12, lmoz12a)
```

```
## Analysis of Variance Table
##
## Model 1: log(ozone) ~ temp + wind + rad + I(wind^2) + I(rad^2)
## Model 2: log(ozone) ~ I(temp/100) + wind + rad + I((wind/100)^2)
##   Res.Df    RSS Df  Sum of Sq F Pr(>F)
## 1    105 24.707
## 2    105 24.707  0 1.0658e-14
```

# IMPORTÂNCIA DAS VARIÁVEIS

Escalas diferentes: problema

```
coef(lmoz12)
```

```
##    (Intercept)           temp           wind            rad       I(w
##   7.723892e-01   4.193355e-02  -2.211428e-01   7.465764e-03   7.3902
##       I(rad^2)
##  -1.470231e-05
```

```
coef(lmoz12a)
```

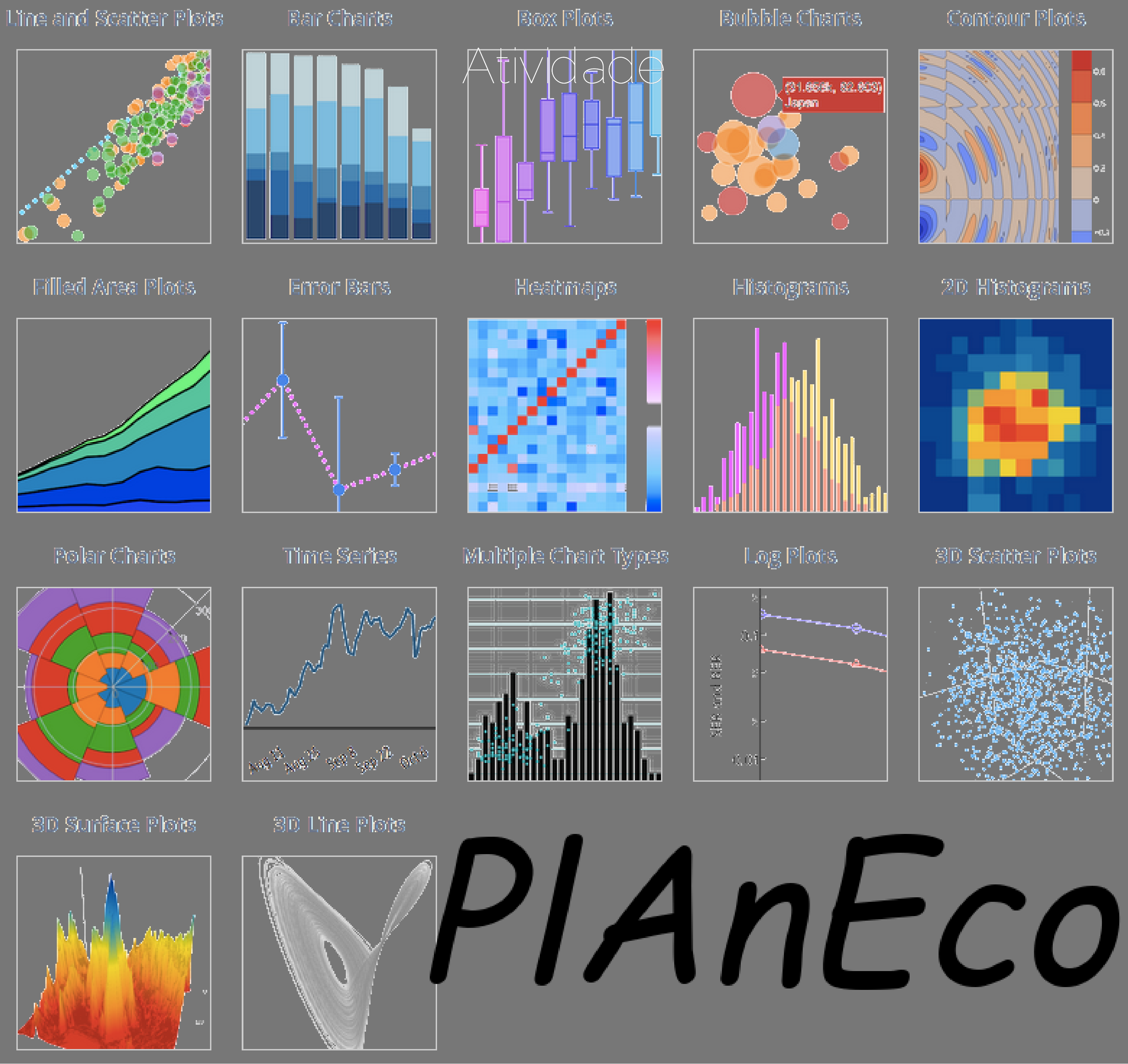
```
##      (Intercept)      I(temp/100)             wind              rad
##     7.723892e-01     4.193355e+00    -2.211428e-01     7.465764e-03
## I((wind/100)^2)        I(rad^2)
##     7.390204e+01    -1.470231e-05
```

# Rescalonando os coeficientes:

```
tempR <- scale(ozo$temp)
windR <- scale(ozo$wind)
radR <- scale (ozo$rad)
## modelo rescalonado
lmoz12R <- lm(log(ozone) ~ tempR + windR + radR + I(radR^2)+
  I(windR^2), data = ozo)
```

# Modelo rescalonado

```
##
## Call:
## lm(formula = log(ozone) ~ tempR + windR + radR + I(radR^2) +
##     I(windR^2), data = ozo)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.85551 -0.25578  0.00248  0.31349  1.16251
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.44421    0.07836  43.951  < 2e-16 ***
## tempR        0.39963    0.05944   6.723 9.52e-10 ***
## windR       -0.26425    0.05688  -4.646 9.86e-06 ***
## radR         0.18520    0.05268   3.516 0.000649 ***
## I(radR^2)   -0.12216    0.05595  -2.183 0.031246 *
## I(windR^2)   0.09362    0.03274   2.859 0.005126 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4851 on 105 degrees of freedom
## Multiple R-squared:  0.7004, Adjusted R-squared:  0.6861
## F-statistic:  49.1 on 5 and 105 DF,  p-value: < 2.2e-16
```


Line and Scatter Plots
Bar Charts
Box Plots
Bubble Charts
Contour Plots
Filled Area Plots
Error Bars
Heatmaps
Histograms
2D Histograms
Polar Charts
Time Series
Multiple Chart Types
Log Plots
3D Scatter Plots
3D Surface Plots
3D Line Plots
Atividade
PlAnEco